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APPROXIMATE METHOD FOR CALCULATING A TURBULENT BOUNDARY LAYER IN THE PRESENCE OF A POSITIVE PRESSURE GRADIENT

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Symbols

u -- velocity;
ρ -- density;
δ^* -- width of displacement;
δ^{**} -- width of pulse loss;
θ -- width of energy loss;
M -- Mach number;
r -- radius;
μ -- dynamic viscosity;
c_p -- heat capacity at constant pressure;
R_{δ_0} -- Reynolds number calculated according to the thickness of the boundary layer of the initial cross section;
P -- Prandtl number;
p_1 -- static pressure at the point of initial interaction;
p_2 -- static pressure at the "plateau" of pressures;
p_0 -- stagnation pressure;
T_0 -- stagnation temperature;
I -- enthalpy;
T_r -- recovery temperature;
T_w^0 -- temperature factor;
H -- shape factor
r_1 -- restoration coefficient;

$$T_w^0 = \frac{T_w}{T_e}, \qquad H = \frac{\delta^*}{\delta^{**}}, \qquad \Gamma = \frac{\delta^*}{\rho_1 u_1^2}\frac{dp_1}{dx}$$

Subscripts: 0 -- initial cross section of boundary layer, 1 -- parameters taken on the boundary of the boundary layer, w -- parameters taken for the temperature of the wall, * -- parameters related to flow on a flat plate for $\Gamma = 0$, and prime indicates differentiation with respect to x.

APPROXIMATE METHOD FOR CALCULATING A TURBULENT BOUNDARY LAYER IN THE PRESENCE OF A POSITIVE PRESSURE GRADIENT

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ABSTRACT. Proposed herein is a uniparametric integral method for calculating a turbulent boundary layer in the presence of a positive pressure gradient which makes it possible to calculate the friction, heat flow, and thickness of the layer both in front of the break-away point and in some zone behind the break-away point.

As in the method of V. S. Avduyevskiy [1], the experimental relations for /64[1] friction coefficient, Stanton number, and shape factor H on Mach number M, Reynolds number R, temperature factor T_w^0, etc., are used here, but in contrast to [1], it is assumed that the function of D is proportional to friction, the function of F is proportional to the heat flow, and the shape factor H is a function of the parameter of pressure gradient Γ, while the functions $D(\Gamma)$, $F(\Gamma)$, and $H(\Gamma)$, are found by processing experimental data. This method makes it possible to calculate quite easily the flow in the region of interaction of a turbulent boundary layer with rapid changes in compression in a manner similar to the calculation of the interaction with a laminar layer as considered in [2].

1. We will write the integral equations of motion and energy for an axiometric turbulent boundary layer in a compressed gas as follows:

$$\frac{dY}{dx} + a_1 Y H^0 = b_1 D^0 \qquad \frac{dZ}{dx} = EF^0 \qquad \left(H^0 = \frac{H}{H^*},\ D^0 = \frac{D}{D^*},\ F^0 = \frac{F}{D^*}\right) \tag{1.1}$$

By following the method outlined in [1], we will write

[1] Numbers in the margin indicate pagination in the foreign text.

$$Y = (r\varphi_1 u_1^2 \delta^{**})^{1/4}, \qquad Z = r\rho_1 u_1 (I_0 - I_w)\delta^{**}\beta$$

$$a_1 = \frac{5}{4} H^* \frac{u_1'}{u_1}, \qquad b_1 = \frac{5}{4} r^{1/4} \rho_w \mu^{1/4} u_1^{1/4} D^*$$

$$H^* = \frac{9}{7}\left[\frac{T_w}{T_0}\left(1 + \frac{\gamma - 1}{2} M^2\right)\right] + \frac{\gamma - 1}{2} M^2$$

$$D^* = 0.0129 (T_w^0)^{0.43}\left(1 + \frac{\gamma - 1}{2} r_1 M^2\right)^{0.137}$$

$$E = rD^* P^{-n/m}\left(\frac{I_e - I_w}{I_0 - I_w}\right)^{1/6} \beta^{1/6} R_\theta^{-1/4} g \rho_w u_1 (I_e - I_w)$$

$$\beta = \theta / \delta^{**}, \qquad r_1 = 0.89$$

2. We will now examine the flow in the region of the interaction of a turbulent layer with rapid jumps in compression when the temperature factor T_w^0 is close to unity and we will assume that the thicknesses of the heat and dynamic layers are about the same (Figure 1). Under the effect of a rapid change in compression the perturbation of pressure is transmitted upward through the flow along the boundary layer and the pressure distribution on the wall becomes quite smooth. The boundary layer cannot maintain large pressure gradients, and at some value Γ the boundary layer will break away. The pressure will reach a plateau and the values of Γ will be reduced in the region of the interrupted flow. We will assume that break-away takes place at the point of the maximal value $\Gamma = \Gamma_M$ on the surface of the body (experiments for a nearly heat-insulated surface have confirmed this assumption). We will then make the following notation: $\Gamma^0 = \Gamma/\Gamma_M$, and we will attempt to determine the form of the functions H^0, D^0, and F^0 from Γ^0. We assume that these functions depend on the numbers M, R, γ, P, and T_w^0, just as for $\Gamma = 0$. For the purpose of determining $D^0(\Gamma^0)$, we used the experimental data found in [3], in which the profiles of velocity in the region of positive pressure gradients are given for a compressed turbulent boundary layer. The function formulated $D^0(\Gamma^0)$ is illustrated in Figure 2.

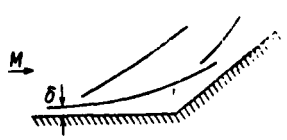

Figure 1

/65

We will further assume that the pressure distribution in the region of the interaction is determined by the distribution of the thickness of displacement δ^* and is approximately described by the equation for the Prandtl-Mayer flow

$$\frac{d^2\delta^*}{dx^2} = \frac{\sqrt{M^2 - 1}}{\gamma p M^2}\frac{dp}{dx} \qquad (2.1)$$

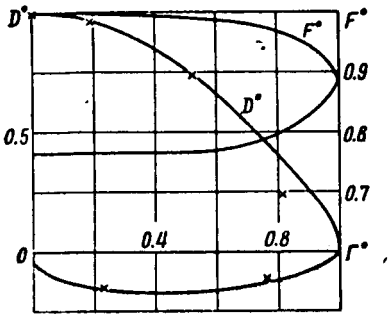

Figure 2

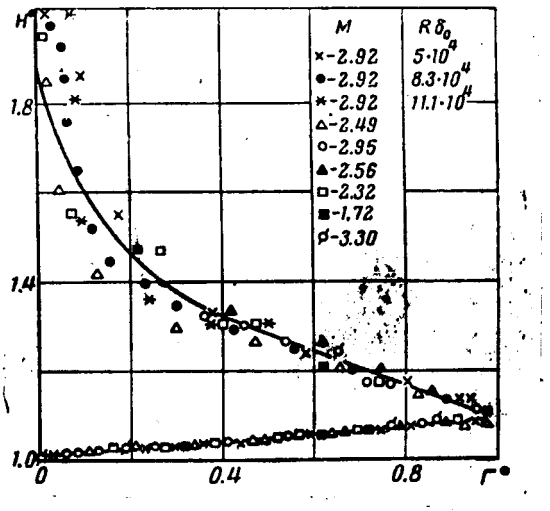

Figure 3

In order to determine $H^0(\Gamma^0)$ it is necessary to solve equations (1.1) and (2.1) jointly. For this purpose we will need to know the pressure distribution through the region of interaction and the values δ_0^*, δ_0^{**}, $(d\delta^*/dx)_0$, and M_0 in the initial cross section x_0. All these values were taken from the experiments in [3, 4]. Then, after solving equations (1.1) and (2.1) step by step with respect to x and computing H_0 and Γ_0 at each step, we constructed the curves of $H^0(\Gamma^0)$, which are illustrated in Figure 3 for various values of M_0 and R_{δ_0}. The function $H^0(\Gamma^0)$ for a combined boundary layer can be approximated by the straight line $H^0 = 1 + 0.1\,\Gamma^0$ in the given range of M_0 numbers from 2 to 3.3 and R_{δ_0} from $3\cdot10^4$ to $11\cdot10^4$. According to the data of G. M. Bam-Zelikovich [7], $\Gamma_M = 0.015$ for a noncompressed flow at the point of break-away. The value of Γ_M was equal to about 0.023 in our experiment for the region of interaction of a turbulent boundary layer with rapid changes in compression within the range of M numbers from 2 to 3.3. In order to calculate the function $F^0(\Gamma^0)$, we solved the second equation of system (1.1), in which the value θ_0 was found from experimental data and the value F^0 was selected at each step in such a way that the computed value of heat flow q_w conformed with experimental measurements.

Figure 2 shows the dependences of H^0 and F^0 on Γ^0 for the region of flow of a turbulent boundary layer, both before the point of break-away and after it.

It should be noted that the dependences of H^0, F^0, and D^0 on Γ^0 are very similar, as regards shape, to the analogous dependences for a lamellar layer, which may be calculated by using the results of [5, 6], both for noninterrupted and interrupted flows. /66

3. We can now solve the inverse problem and calculate the flow in the region of interaction of a turbulent boundary layer and check the sensitivity of the computed values for pressure, heat flow, friction, and thickness to the errors of the experimentally determined dependences of H^0, F^0, and D^0 on Γ^0. We calculate only that part of the flow from the initial interaction to the region of the "plateau" of the pressure. For the solution of this problem we use the following equation system:

$$\frac{d\delta^*}{dx} = \varphi, \qquad \frac{d\varphi}{dx} = \frac{dp}{dx}\frac{\sqrt{M^2-1}}{\rho_1 u_1^2}, \qquad \frac{d\bar{Z}}{dx} = EF^0$$

$$\frac{dp}{dx} = \Gamma\frac{\rho_1 u_1^2}{\delta^*} \qquad \frac{dY}{dx} + a_1 H^0 Y = b_1 D^0 \tag{3.1}$$

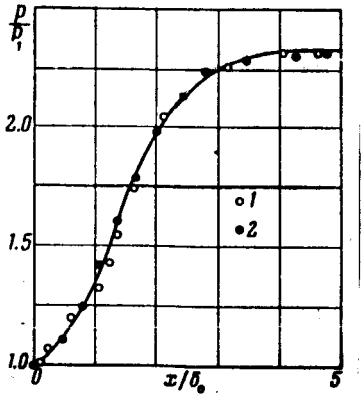

Figure 4

In contrast to the calculation method found in [2] for a lamellar layer, the system of equations for calculating the flow in the region of interaction clearly includes the equation $dp/dx = \Gamma\rho_1 u_1^2/\delta^*$, which makes it possible to eliminate iterations and greatly simplifies the calculation. The dependences of H^0, F^0, and D^0 on Γ^0 complete the equation system. It is assumed that the turbulent layer in front of the region of interaction is divided into sheets ($H^0 = F^0 = D^0 = 1$, $\Gamma^0 = 0$) and δ_0^*, δ_0^{**}, M^0 and p^0 are known in the initial cross section x_0. Furthermore, the "critical" ratio of pressure in the region of the "plateau" to that in the initial part of the break-away zone, p_2/p_1, is assumed to be known. Then, by solving the boundary problem for equation system (3.1) by adjustment, it is possible to find the value ϕ_0. Figure 4 shows an example of the calculation of the pressure distribution in the break-away zone for $M_0 = 2.95$. Here 1 is the experiment in [4], 2 is the calculation based on the method of our experiment. The results of the calculations indicate that in the given range of M and R_{δ_0} numbers the functions $H^0(\Gamma^0)$, $F^0(\Gamma^0)$, and $D^0(\Gamma^0)$ are universal with sufficient accuracy and can be used for calculating flows of a turbulent boundary layer in the presence of a positive pressure gradient.